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Ahmet H Ertas[1] and Fazil O Sonmez[2]

Abstract

Composite structures are usually subjected to fluctuating loads in service leading to fatigue failure. Because it is one of the main failure modes, fatigue behavior of composites has been extensively studied to be able to design fatigue-resistant composite structures. However, little attention has been paid to their design optimization under fatigue loading. In this study, a methodology is proposed to find the optimum fiber orientation angles of composite laminates under various in-plane loads to achieve maximum fatigue life. Fawaz–Ellyin's model is used to predict the fatigue life of the laminates. A variant of simulated annealing algorithm is used as the search algorithm in the optimization procedure. A number of problems are solved to demonstrate the effectiveness of the proposed method.

Keywords

Multidirectional laminates, fatigue, life prediction, global optimum design, simulated annealing

Introduction

Composites are used in many structural applications such as airplanes, ships, sporting goods, due to their superior properties like strength-to-weight or stiffness-to-weight ratios. In addition, composites are less sensitive to fatigue than metals. As a further advantage, composite structures offer great flexibility in design by allowing change of the material system in many different ways.

The durability of a structure is defined as its ability to maintain its mechanical performance through its service life. Durability is an indication of safety as well as cost considering that maintenance and repair expenses throughout the service life depend on the durability of the structure. In composites, as in metals, damage creation and growth are mainly due to fatigue under typical service loading leading to the loss of structural integrity.[1]

Through an optimal selection of fiber and matrix materials, fiber orientations, stacking sequence, and lamina thickness, one may significantly increase fatigue life of a composite structure and thus its durability. Although there are numerous studies on design optimization of composite structures subjected to static loading, very few published studies exist on optimal design of composites susceptible to fatigue failure.[2,3] Adali[2] optimized a symmetric angle-ply laminate under in-plane cyclic loads and determined the maximum failure load. Walker[3] presented a procedure to minimize the thickness of laminated composite plates under cyclic loads by using a specific limit for fatigue life as a constraint. In the previous studies, the researchers used fatigue models that were valid only for limited laminate configurations and specific loading conditions. Usually, more general layups and loading conditions need to be considered in design optimization for typical applications.

In order to maximize the fatigue life of a composite laminate, one should first be able to accurately calculate its fatigue life for any arbitrary configuration. This requires a reliable fatigue assessment model. The extensive research on composite materials[1,4–21] revealed that their fatigue behavior was more complex than that of metals. In a metal, fatigue damage occurs through initiation and propagation of sharp cracks, which can be

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reasonably well predicted through the conventional fatigue assessment models. Fatigue damage in composite materials, on the other hand, may result from several mechanisms like fiber-splitting, fiber-breakage, transverse micro-cracking, matrix cracking, fiber/matrix debonding, void growth, and delaminations or a combination of them.[4]

In this study, various fatigue assessment models were examined and Fawaz and Ellyin's model[21] was found to be the most suitable one for an optimization study. In laminate design, materials of fibers and matrix, fiber orientations, and lamina thicknesses are considered as design variables. In this study, just fiber orientations are considered as optimization variables. This means that the results are optimum for the preselected material and lamina thicknesses. However, by repeating the optimization process for different materials and lamina thicknesses, one may obtain optimum designs for different cases.

Optimizing complex configurations requires the use of a global search algorithm. Local search algorithms such as the ones used in the previous studies may not find the best possible design in optimization problems having complex design domain. Considering that numerous local optimum designs exist in composite optimization problems, a variant of simulated annealing (SA) is used to search the globally optimal designs.

Fatigue life assessment models for composite materials

Fatigue life prediction models for composite materials are mainly classified in five categories;[6,22] cumulative or progressive damage,[5,7,19,23–25] probabilistic,[8,10,23,26–28] phenomenological/empirical,[8,9,15,20,21,29–35] artificial neural network based,[36,37] and micromechanics[5,22,38] models.

In cumulative or progressive damage models, the extent of material deterioration, which may range from the initiation of damage up to the final catastrophic failure, is estimated corresponding to a given load fluctuation. Fatigue life of composites shows a larger scatter in comparison to metals due to formation of defects and non-uniformities in the microstructure during the manufacturing process.[26,33] Probabilistic models provide a statistical relationship between loading and the corresponding fatigue life. Phenomenological/empirical models estimate the fatigue life due to constant amplitude loading based on experimental data without making any assumption regarding the micro mechanisms leading to fatigue failure. In artificial neural network-based models, the relation between applied load and fatigue life is obtained through a neural network system previously trained using existing empirical data. Micromechanical models, on the other hand, attempt to describe the fatigue response of composites based on the fatigue behavior of the constituent materials.

Although many fatigue life prediction models[5,7,8–10,13–15,20,21,23–39] were proposed for composites, most of them are not suitable to use in general design optimization studies because of their restricted applicability. The existing models are usually applicable only to, or validated only for, specific layups[10,14,15,20,21,26,29–33,35–37,39] or particular lamina thicknesses.[5,14,34,36] Most of the models were validated for uniaxial tension, some of them[7,20,21] for biaxial tension, some for shear loading,[26] and some[23,30] for bending. Many of them[9,10,13,14,23,31,32,34–36,39] are validated for just one type of material. Some of them[8,11,15,20,21,24,28,34,36] account for the effect of R-ratio.

Besides the general applicability, ease of use and requirement of few empirical constants are essential for a model to be used in a design optimization study. In these respects, Fawaz and Ellyin's model[21] seems to be a promising model considering its applicability to various in-plane loading states and arbitrary fiber orientations requiring only two empirical constants. However, this model has not yet been validated for multiaxial laminates with arbitrary layup configurations. In this study, first, the validity of the model for these laminate configurations is investigated. Secondly, applicability of the first-ply-failure approach as opposed to progressive failure is studied. After these investigations, a methodology for design optimization of composite laminates under fatigue loading is proposed and then applied.

Fawaz and Ellyin's parametric fatigue life prediction model

The fatigue life of a fiber-reinforced laminate depends primarily on the type of constituent materials, fiber orientation angle, θ, the range of cyclic stress, $\Delta\sigma_{ij}$, and R ratio, $\sigma_{\min}/\sigma_{\max}$. Fawaz and Ellyin's model[21] accounts for all these factors using only basic and easily obtained material properties.

Consider a continuous fiber-reinforced lamina subjected to in-plane stresses, as shown in Figure 1. Fawaz and Ellyin assume that a semi-log linear relationship exists between the range of cyclic stress and the fatigue life as the following:[21]

$$\Delta\sigma_{xx} = m\log(N) + b \tag{1}$$

where $\Delta\sigma_{xx}$ is the alternating normal stress along the x direction $(\sigma_{\max} - \sigma_{\min})/2$, m, and b are parameters having values that depend on the material, the orientation angle, θ, and the stress state, and N is the corresponding number of cycles to failure.

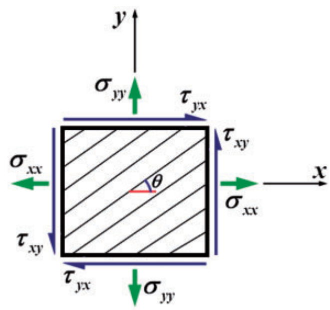

Figure 1. A representative geometry of a unidirectional lamina showing stresses and fiber orientation angle.

First, the relation is empirically established for a chosen reference direction, θ_r, as the following:

$$\Delta\sigma_{xx} = m_r \log(N) + b_r \quad (2)$$

where m_r and b_r are the constants found by curve fitting of the experimental data, i.e. the measured fatigue lives corresponding to different levels of alternating stress applied to a lamina oriented with θ_r. The parameters in equations (1) and (2) are related by

$$\begin{aligned} m &= f(a_1, a_2, \theta)\, g(R) m_r \\ b &= f(a_1, a_2, \theta) b_r \end{aligned} \quad (3)$$

where f and g are non-dimensional functions given by

$$f(a_1, a_2, \theta) = \frac{S_{xx}(a_1, a_2, \theta)}{S_{xxr}(a_{1r}, a_{2r}, \theta_r)} \quad (4)$$

$$g(R) = \left(\frac{\sigma_{\max}}{\sigma_{(\max)r}}\right)\left(\frac{1-R}{1-R_r}\right) \quad (5)$$

where S_{xx} and S_{xxr} are the level of σ_{xx} at which static failure occurs for the actual and reference configurations, respectively, f accounts for the difference in the static strength between the actual and the reference configurations due to the difference in the fiber angles (θ and θ_r) and biaxial stress ratios (a_1 and a_2), while g explains the difference in the R-ratio. These are given by

$$a_1 = \sigma_{yy}/\sigma_{xx} \quad (6)$$

$$a_2 = \tau_{xy}/\sigma_{xx} \quad (7)$$

$$R = \sigma_{\min}/\sigma_{\max} \quad (8)$$

In order to calculate the strength correlation factor, f, the magnitude of σ_{xx} at which static failure occurs is required. For this purpose, a static failure criterion, Tsai–Hill, is used. In Fawaz and Ellyin model, the model parameters are found by conducting fatigue tests on laminates having the reference angle, θ_r. After obtaining the empirical relation between stress and life for this reference fiber orientation, the relations for other angles are obtained by using a static failure criterion assuming that the change in fatigue strength is correlated with the change in static strength. They obtained this correlation with Tsai–Hill static failure criterion.

Experimental correlation of the model

Fawaz and Ellyin validated their model for unidirectional glass/epoxy, graphite/epoxy, and boron/aluminum laminates under uniaxial loading and also bi-directional woven laminates under biaxial loading with different biaxial stress ratios by comparing the predictions of their model with the experimental data available in the literature for these types of laminates for different load levels and angular orientations. The agreement was satisfactory.

In this study, the model is applied to multidirectional laminates that consist of laminae having different fiber orientations. First, the fatigue life of a lamina is assumed to depend on its local stress state not directly on the global loading conditions on the laminate. Secondly, some of the interactions between plies are neglected. Residual stresses may develop during manufacturing of laminated plates due to discrepancy in the thermal expansion coefficients of laminae. Progression of cracks may be impeded under the constraining effect of the adjacent laminae. These have some bearing on the fatigue life; however, these factors are not considered to be the dominating factors for the problem considered in the present study. The fatigue life of a lamina in a laminate is calculated the same as a unidirectional laminate having the same thickness and orientation angle under the same stress state. Some interactions like stress concentrations at the free edges arising due to different fiber orientations of stacked plies are taken into account by imposing constraints on the optimization. In order to validate this approach, the results of the model are compared with the measured fatigue lives of multidirectional laminates reported by Rotem and Nelson[9] on graphite/epoxy laminates. For the reference values, m_r and b_r, the fatigue life (S-N) curve for $[\pm45]_{4s}$ laminate is used ($m_r = -23.38$, $b_r = 189$). Neglecting the free edge effects and considering that the stress state in $+45°$-laminae is about the same as that of $-45°$-laminae except that shear stresses reverse sign, the life of $[\pm45]_{4s}$ laminate is assumed to be close to that of $[45_8]_s$.

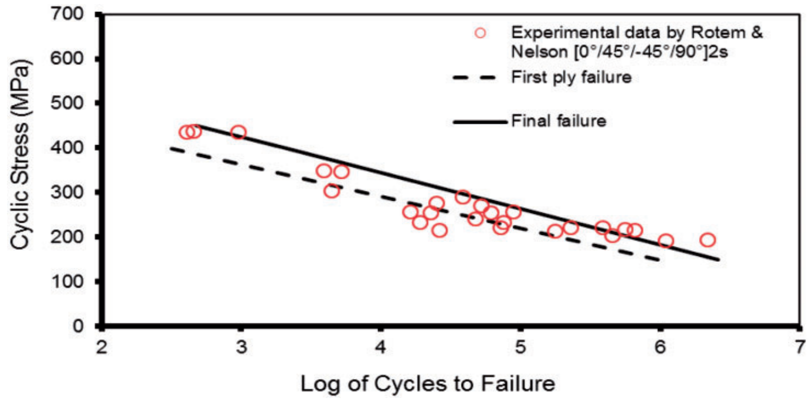

Figure 2. Comparison of the first-ply and last-ply failure predictions of Fawaz and Ellyin's model[21] with the results of Rotem and Nelson[39] on quasi-isotropic glass/epoxy laminates.

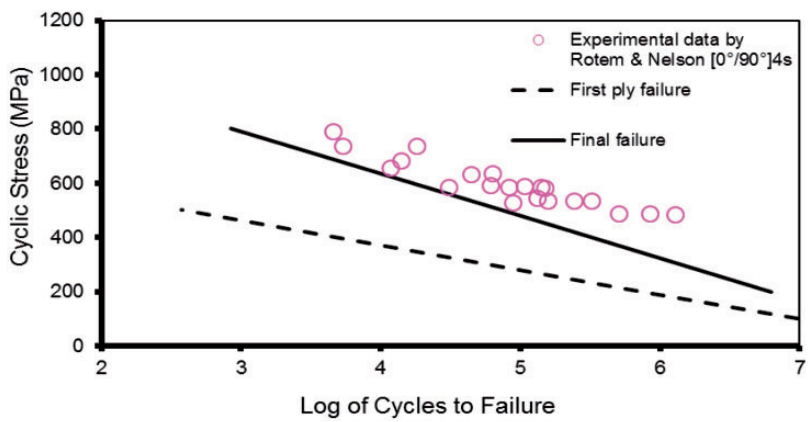

Figure 3. Comparison of the first-ply and last-ply failure predictions of Fawaz and Ellyin's model[21] with the results of Rotem and Nelson[39] on cross-ply glass/epoxy laminates.

Figure 2 presents a case in which the first and last failure loads are different. Rotem and Nelson[39] provided the fatigue lives of quasi-isotropic and symmetric, $[0/45/-45/90]_{2s}$, specimens subjected to different levels of alternating stress. In order to calculate the first-ply failure life, the stress state in each lamina is found using classical lamination theory; then the fatigue life of each lamina is calculated using Fawaz and Ellyin's model; the lowest, which is that of 90°-lamina, is taken as the first-ply failure life. After 90°-laminae fail, the remaining laminae continue to transmit the load. In order to calculate the remaining fatigue life, a progressive approach is used. After the 90°-laminae fail, the load is redistributed among the surviving laminae. For this reason, the stress states are recalculated and the fatigue lives are reevaluated. Because, after each lamina failure, stress amplitudes change, Miner's rule is used to determine the fatigue life in the remaining laminae. This procedure is repeated for several load levels and the first and final fatigue lives versus average stress are plotted in Figure 2. For this laminate configuration, no significant difference exists between the first-ply and last-ply failure lives and the predictions are within the scatter of the empirical data. The reason is that although 90°-laminae are quite weak against normal stresses in the x direction compared to 0°-laminae (74 MPa versus 1630 MPa strength), they are subjected to much lower stresses (20 MPa versus 250 MPa). Accordingly, 90°-laminae do not fail prematurely.

Figure 3 presents a comparison of the first-ply and last-ply failure predictions with the experimental data provided by Rotem and Nelson.[9] The first-ply failure predictions turn out to be greatly conservative for this configuration. Only a progressive failure approach can represent the actual failure behavior for such layup configurations. One recognizes that if the fatigue lives of the individual laminae in a laminate are close to each other like the one shown in Figure 2, the first-ply failure approach gives acceptable predictions; but if a huge difference exists between the lives of laminae like the case in Figure 3, the first-ply failure approach yields highly conservative results. This means that it can be used in a design optimization study where layer thickness and orientations are optimized to maximize the fatigue strength. The layers in an optimized laminate will then have more or less the same fatigue strength; the first and the last-failure lives will then be the same. In view of that, the first-ply failure approach is adopted in this study, and this assumption is validated after the results are obtained.

One should note that the model does not account for the effects of frequency, temperature, and moisture on fatigue life. Therefore, the predictions are valid only if the empirical reference S-N curve, equation (2), is obtained in the same environmental conditions.

Problem statement

The problem considered in this study is to design laminated composites subjected to in-plane loads for maximum fatigue life. The number of distinct laminae, n, and their thickness, t_o, are given. The orientation angle, θ_k, in each lamina is to be determined in the design process. Accordingly, the number of design variables is n. The orientation angles take discrete values; in other words, they are chosen from a finite set of angles. According to the manufacturing precision, the interval between the consecutive angles may be 15°, 10°, 5°, or even smaller.

Formulation of the objective function

Considering that after design optimization, fatigue strengths of laminae comprising the laminate will be about the same and thus they are expected to fail simultaneously, the first-ply failure approach is adopted in the present design optimization study rather than a progressive failure approach. Then, the laminate is considered to have the same fatigue life as the lamina that has the shortest fatigue life, N.

In laminate design, the number of contiguous plies of the same orientation is limited to prevent progression of matrix damage through the thickness and thus avoid large matrix cracks; the maximum thickness is experimentally determined and it is usually taken as four plies for CFRP laminates under static loading.[40] Some experimental studies[41] suggest increased matrix crack growth for laminates with laminae containing more than two plies. As another failure mode, edge delaminations tend to increase if the difference between the angles of two consecutive laminae exceeds 45°.[40] These factors are taken into account in the present optimization procedure by imposing constraints and adding penalty values to the objective function whenever these constraints are violated. Whenever a new configuration is generated by the search algorithm, these requirements are checked; if the difference between the fiber angles of two consecutive laminae, $\Delta\theta$, is greater 45°, a penalty value equal to $\Delta\theta - 45$ is added. If the number of plies in a lamina, n_k, is larger than two, a penalty value equal to $10(n_i - 2)$ is added. If the difference between the fiber angles of consecutive laminae is less than 5°, the sum of the number of plies in those laminae is also constrained not to exceed two. Considering these constraints, the objective function is formulated as

$$f = -\log(N) + c_1 \sum_{i \in I} 10\,(n_i - 2) + c_2 \sum_{j \in J} \left(\Delta\theta_j - 45\right) \tag{9}$$

where n_i is the number of consecutive plies having the same orientation, I is the set of laminae in which the contiguity constraint is violated, $\Delta\theta_j$ is the difference between the fiber angles of two consecutive laminae, J is the set of laminae in which the angle-difference constraint is violated, c_1 and c_2 are weighing factors. Because the search algorithm is constructed to minimize the objective function, the negative of the fatigue life of the laminate, N, is taken in the objective function. Besides logarithm of N is taken so that the order of magnitudes of the terms in the objective function are about the same.

In order to determine N, fatigue life of each lamina is calculated using equation (1) and the smallest of them is chosen as the fatigue life of the laminate, N.

The optimization procedure proposed in this study does not make use of the endurance limit of the material. A laminate configuration is considered to be more fatigue-resistant than another if the fatigue life estimated by the fatigue model is longer than that of the other even if the applied cyclic stress is less than their endurance limits and actually they both have infinite fatigue life. Here we assume that the former will have a longer life in a harsher loading condition.

Optimization methodology

Optimization algorithm

In this study, a variant of SA algorithm is used.[42] The search algorithm is presented in Figure 4. The algorithm is similar to direct search simulated annealing (DSA)[43] in that a set of current configurations rather than a single current configuration is maintained during the optimization process. Accordingly, unlike the standard SA algorithm where only the neighborhood of a single point is searched, the present algorithm

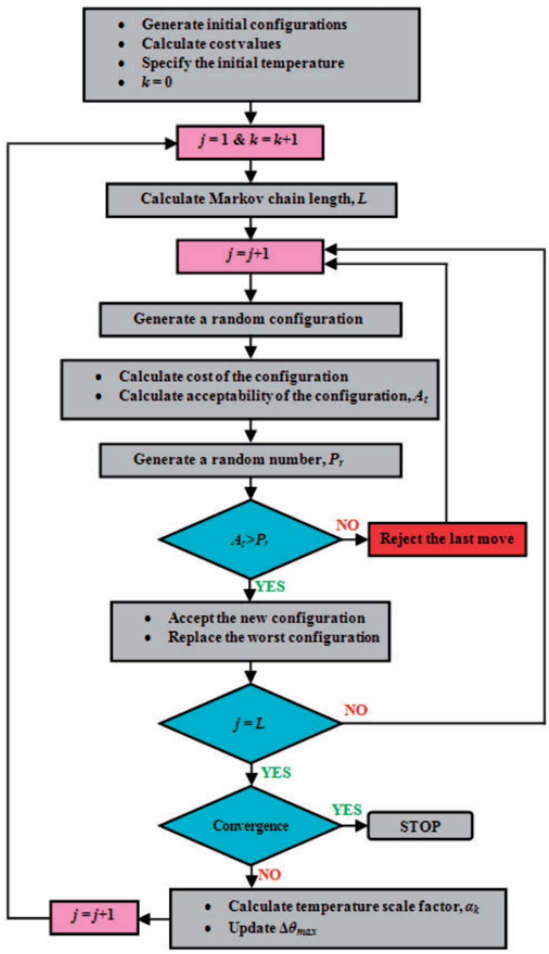

Figure 4. The main optimization procedure.

searches the neighborhood of all the current points in the set. At the start of the optimization process, initial configurations are randomly created within the design domain by randomly selecting values for the design variables. The number of initial configuration, M, is equal to $8(n+1)$, where n is the number of design variables as mentioned before. The design variables are the orientation angles of the fibers in the laminae, θ_i. In order to generate an initial laminate configuration, values among (-90, $-90+\phi, \ldots,$ -2ϕ, $-\phi$, 0, ϕ, $2\phi, \ldots,$ $90-\phi$, 90) are randomly chosen for the lamina angles, θ_i. Here ϕ is the interval between consecutive angles, which may be 1°, 5°, 10°, 15°, 30°, 45°, or 90° according to the manufacturing precision. After the initial laminate configurations are randomly chosen, their objective function values are calculated. SA requires random generation of a new configuration in each iteration. For this purpose, a configuration in the neighborhood of one of the current configurations is randomly generated as follows: First, one of the current configurations is randomly chosen. Then, random differences are introduced to the fiber orientation angles.

$$\theta'_k = \theta_k + r\,\Delta\theta_{\max} \tag{10}$$

where θ_i is the fiber orientation angle in the i^{th} layer of the randomly chosen current configuration; θ'_i is the fiber orientation angle of the i^{th} lamina in the newly generated configuration; r is a randomly chosen real number within $[-1, 1]$; $\Delta\theta_{\max}$ is the maximum variation that may be introduced to θ_i. θ'_i is rounded to the nearest discrete angle along which the fibers can be oriented. The lower and upper limits for θ'_i are -90° and 90°, respectively. If a number greater than 90 is generated for θ'_i, 180 is subtracted from this number. If it is less than -90, 180 is added. Then, the structural analysis of the laminate is carried out using the classical lamination theory to determine the stress state generated due to the maximum load. Since the structure is assumed to be linear, the stresses corresponding to the minimum load are obtained by reducing the calculated stresses by the minimum-to-maximum-load ratio. Using the calculated values of the stress components, $\Delta\sigma_{xx}$, $\Delta\sigma_{yy}$, and $\Delta\tau_{xx}$, the fatigue life of each lamina is estimated using equation (1). Then, the minimum value is chosen as the fatigue life of the laminate. Acceptability of a newly generated trial configuration, A_t, depends on the value of its cost, f_t, which is calculated by

$$A_t = \begin{cases} 1 & \text{if} \quad f_t \le f_h \\ \exp((f_h - f_t)/T_k) & \text{if} \quad f_t > f_h \end{cases} \tag{11}$$

where f_h is the cost (objective function value) of one of the high-cost current configurations; T_k is the temperature parameter for the k^{th} Markov chain. Iterations during which the value of the temperature (or control) parameter, T_k, is kept constant are called k^{th} Markov chain (or inner loop). According to equation (11), every new design having a cost lower than the cost of the worst design is accepted. But, if the cost is higher, the trial configuration may be accepted depending on the value of A_t. If it is greater than a number randomly generated between 0 and 1, P_r, the trial configuration is accepted, otherwise rejected. The number of iterations, L, executed in a Markow chain is calculated according to the formula provided by Ali et al.[43]:

$$L_k = L + L\left(1 - e^{f_\ell - f_h}\right) \tag{12}$$

Here

$$L = 10n \tag{13}$$

where n is the number of design variables, which is equal to the number of distinct lamina angles; f_ℓ is the value of the best current configuration, which has lowest objective function value.

The temperature parameter, T, controls the convergence of the optimization process just like the temperature controls micro-structural changes in the physical annealing process. At the start, the value of the temperature parameter is chosen to be so large that almost any arbitrary configuration is accepted. This is because in order to prevent getting stuck into a local minimum at early stages of the optimization and search a large design space, the probability of acceptability must be high. This stage resembles melting in the physical annealing where atoms can rearrange themselves freely. After all the iterations in an inner loop are executed, the temperature parameter, T, is reduced, a new inner loop begins. The reduction scheme is as follows

$$T_k = \alpha_k T_{k-1} \tag{14}$$

where T_{k-1} is the temperature parameter in the $(k{-}1)^{\text{th}}$ Markov chain and α_k is temperature reduction factor. As equation (11) implies, when T is decreased, the probability that a configuration with a higher cost is accepted becomes lower. At low values of temperature parameter, acceptability becomes low; thus at low temperatures, acceptance of worse configurations is unlikely, just as the atoms become stable, and do not tend to change their arrangements.

In DSA, if a new configuration is accepted, it replaces the worst configuration. The replacement scheme of DSA has one drawback that becomes especially apparent with a high number of design variables. For instance, if one optimizes a laminate with 16 distinct lamina angles using DSA, the number of current configurations will be $8(16+1)=136$. At the initial stages of the optimization process, the current configurations quickly gather around local minimums except the worst one, which is frequently updated by higher-cost configurations at high temperatures. Because new configurations are generated in the neighbourhood of the current ones, search becomes restricted to a small segment of the feasible domain except for the trials in which the new configuration is generated around the worst current configuration, which are very infrequent. In order to remedy this, the replacement scheme of DSA was modified. The current configurations are ordered with respect to their objective function value. If a new configuration is accepted, it replaces a current configuration randomly chosen among $(n+1)$ worst configurations instead of the worst one. Thus, $7(n+1)$ of the current configurations having low cost remain in the set unless better ones are found through iterations. The others, on the other hand, may be replaced by higher cost configurations with a probability depending on the temperature parameter. In this way, $(n+1)$ configurations become dispersed in the feasible domain at high temperatures and thus a thorough search of the feasible domain can be achieved. In the present version of the algorithm, the objective function value of the best of the worst $(n+1)$ current configurations is used for f_h in equation (11).

In order to find the globally optimal design, one should be able to search a large solution domain. For this reason, instead of giving small perturbations to the current configuration to obtain a new configuration in its near neighborhood, one should allow a large variance in the current configurations. For this reason, the magnitude of $\Delta\theta_{\max}$ is taken as 50°. This means that the neighborhood of a current configuration where a new configuration is generated is initially quite large. In this way, the solution domain can fully be searched. This can also be considered as a logical consequence of simulating the physical annealing process, where mobility of atoms is large at high temperatures, and thus the probability that atoms may form a quite different configuration is high. In the present search, algorithm unlike DSA, the magnitude of $\Delta\theta_{\max}$ is not kept constant. $\Delta\theta_{\max}$, thus variation in θ_k, is reduced as the temperature parameter is decreased; but the reduction scheme of the search algorithm does not directly depend on temperature. If there is no improvement on the worst of the best $7(n+1)$ current configurations during a Markow chain, $\Delta\theta_{\max}$ is reduced by multiplying with a factor to make the searched region smaller and thus increase the likelihood of finding a better design.

$$\Delta\theta'_{\max} = c\,\Delta\theta_{\max} \tag{15}$$

where $c=0.999$. Reduction of the maximum variation in the optimization variables is a feature of the present search algorithm that is different from DSA: Here, there is a similarity to the physical annealing process, where the mobility of atoms decreases as the temperature is lowered. Reduction in the temperature should be slow enough to allow time-dependent micro-structural changes to occur and to reach equilibrium. In SA, non-improvement in the current set implies that further changes are unlikely with the current values of $\Delta\theta_{\max}$. For this reason, a reduction scheme different from SA or DSA algorithms is adopted for the temperature parameter. Instead of reducing T_k by a specific ratio, it is made dependent on the reduction in $\Delta\theta_{\max}$, because it is a better indication of equilibrium at a specific temperature level. The temperature reduction factor, α_k in equation (14), is calculated as

$$\alpha_k = \begin{cases} \alpha_{\max} & \text{if } L_a^k/L^k < [(\Delta\theta_{\max})/(\Delta\theta_{i\max})] + 0.01 \\ \alpha_{\min} & \text{else} \end{cases} \tag{16}$$

Table 1. The optimum lay-ups of laminates with a configuration of $[\alpha_4/\beta_4/\gamma_4/\theta_4]_s$ and the corresponding fatigue lives for various in-plane normal loads.

Loading ($\times 10^5$ N/m) $N_{xx}/N_{yy}/N_{xy}$	Optimum lay-up sequences	Failure indices (Tsai–Wu)	Failure indices (max. stress)	Fatigue life (cycles)
5/0/0	$[0_{16}]_s$	0.010	0.100	5.633×10^8
5/3/0	$[-38_8/38_8]_s$	0.145	0.419	284,222
5/5/0	$[\theta_4/(\theta+90)_4/\beta_4/(\beta+90)_4]_s$, $[-10_4/80_4/-30_4/60_4]_s$, $[-45_4/45_4/-15_4/75_4]_s, \ldots$	0.262	0.579	11,190
5/7/0	$[49_8/-50_8]_s$	0.355	0.663	1291
5/−5/0	$[0_8/90_8]_s$	2.121	2.892	972,480
5/−7/0	$[0_8/90_8]_s$	1.542	1.884	25,926

where L^k is the number of trials executed in the k^{th} Markow chain and L_a^k is the number of accepted configurations. Accordingly, L_a^k/L^k ratio is a measure of acceptability in a Markow chain. $\Delta\theta_{\max}$ and $\Delta\theta_{i\max}$ are the current and initial maximum variation in the lamina angles, respectively. $\alpha_{\max}$ and $\alpha_{\min}$ are taken to be 0.9999 and 0.99. Through the use of this equation, the value of the temperature parameter is controlled so that acceptability of Markow chains, L_a^k/L^k, become out equal to the ratio of the current and initial maximum variance, $\Delta\theta_{\max}/\Delta\theta_{i\max}$. When the acceptability is higher, temperature parameter is reduced at a faster rate, $\alpha_{\min}$; otherwise at a slower rate, $\alpha_{\max}$. At the initial stages of the optimization, the right hand side of the inequality is close to 1.0; accordingly, the temperature level is kept high such that the ratio of the number of accepted trials to the total number of trials remains close to 1.0. When $\Delta\theta_{\max}$ approaches zero at the end of the optimization process, temperature parameter also approaches zero; acceptability of a new configuration, A_t in equation (11), then comes close to zero. Iterations are continued until the difference between the values of the best and the worst current configurations becomes small.

Numerical results and discussions

Results were obtained for a unidirectional E-glass/epoxy fiber-reinforced material with the material properties given[31] as $E_{11} = 181$ GPa, $E_{22} = 10.3$ GPa, $G_{12} = 7.17$ GPa, $\nu_{12} = 0.28$, $X_t = -X_c = 1235.64$ MPa, $Y_t = -Y_c = 28.44$ MPa, and $S = 37.95$ MPa. The ply thickness is 0.127 mm. The reference values for fatigue life are $m_r = -15.31$ and $b_r = 148.13$ obtained for $\theta_r = 15^\circ$. R ratio is 0.1. The aforementioned design optimization algorithm was applied to laminates having various lay-up configurations and subjected to different types of in-plane loads.

Optimum designs obtained with unconstrained optimization

First, laminates are optimized using the aforementioned optimization procedure to obtain the maximum fatigue life without imposing contiguity and angle difference constraints. Table 1 shows the optimum lay-up configurations for a laminate with four distinct lamina angles, $[\alpha_4/\beta_4/\gamma_4/\theta_4]_s$, subjected to various in-plane normal loads. The table also presents the corresponding fatigue lives and the failure indices calculated according to Tsai–Wu and maximum stress static failure criteria. One should note that the objective function of the unconstrained optimization is not affected by the stacking sequence for in-plane loads. For this reason, alternative stacking sequences are not shown in the tables. For example, the sequence $[0_4/90_4 0_4/90_4]_s$ is shown as $[0_8/90_8]_s$. As the results indicate, all the laminate configurations are safe against static failure. The fatigue life is found to be very sensitive to the level of stress. With a change in stress level, the fatigue life dramatically changes.

Table 2 shows the effect of the existence of shear stress on the optimal designs. The results given in these tables suggest a correlation between the static strength and the fatigue strength. If a laminate is stronger against static loading, it is likely that it is also stronger against fatigue loading. However, this is not a rule. As seen in the tables, there are laminate configurations having a smaller safety factor against static failure, but a longer fatigue life.

If a higher number of design variables are used, the solution domain is enlarged and within the enlarged domain a better optimum design may be found. Table 3 shows the results obtained using 16 distinct fiber angles. For some loading cases, the same design is obtained; but for others, the algorithm finds improved designs.

Table 2. The effect of shear forces on the optimum lay-ups of laminates with a configuration of $[\alpha_4/\beta_4/\gamma_4/\theta_4]$ and the corresponding fatigue lives.

Loading ($\times 10^5$ N/m) $N_{xx}/N_{yy}/N_{xy}$	Optimum lay-up sequences	Failure indices (Tsai–Wu)	Failure indices (max. stress)	Fatigue life (cycles)
0/0/5	$[45_8/-45_8]_s$	0.222	0.346	972,480
5/0/5	$[31_4/32_8/-58_4]_s$	0.290	0.400	283,626
5/2.5/5	$[38_{12}/-52_4]_s$	0.318	0.504	76,212
5/5/5	$[45_{16}]_s$	0.040	0.199	6.700×10^7
5/7.5/5	$[34_8/70_8]_s$	0.105	0.246	1.729×10^6
5/10/5	$[37_8/80_8]_s$	0.184	0.322	84,475
5/5/2.5	$[14_8/76_8]_s$	0.152	0.345	226,282
5/5/7.5	$[-45_4/45_{12}]_s$	0.628	0.685	1186

Table 3. The optimum lay-ups of laminates obtained using 16 distinct lamina angles and the corresponding fatigue lives for various loadings.

Loading ($\times 10^5$ N/m) $N_{xx}/N_{yy}/N_{xy}$	Optimum lay-up sequences	Failure indices (Tsai–Wu)	Fatigue life (cycles)
5/7/0	$[49_5/-49_5/-50_3/50_3]_s$	(0.356/0.356/0.352/0.352)	(1296/1296/1369/1369)
5/−7/0	$[0_7/90_9]_s$	(0.346/0.289)	(106,130/323,617)
5/2.5/5	$[38_{14}/-52_2]_s$	(0.191/0.243)	(2,001,512/576,498)
5/10/5	$[36_7/42/80_8]_s$	(0.187/0.142/0.186)	(85,514/269,161/87,032)
5/5/7.5	$[-45_2/45_{14}]_s$	(0.492/0.566)	(17,029/5504)

Table 4. The effect of shear forces on the optimum lay-ups of laminates with a configuration of $[\alpha_4/\beta_4/\gamma_4/\theta_4]$ and the corresponding fatigue lives.

Loading ($\times 10^5$ N/m) $N_{xx}/N_{yy}/N_{xy}$	Optimum lay-up sequences	Failure indices (Tsai–Wu)	Fatigue life (cycles)
0/0/5	$[-47/-45/-42/-47/-42/-45/-47/-23/22/43/46/48/43/46/48/43]_s$	0.265	171,738
5/2.5/5	$[41/39/36/41/39/36/41/39/36/42/39/37/3/-42/-60/-55]_s$	0.307	118,376
5/5/2.5	$[11/16/11_2/16/11_2/26/71/78_2/73/78_2/73/78]_s$	0.165	160,972
5/5/7.5	$[-46_2/-59/76/47/45/42/47_2/42/45/47/42/45/48/43]_s$	0.606	1642

Table 3 also presents failure indices calculated using Tsai-Wu static failure criterion and fatigue lives of each lamina. After the fiber orientations of the laminate are optimized, static strengths of the laminae nearly become equivalent. The differences may be attributed to the discrete nature of lamina angles and thicknesses. Because fatigue life is very sensitive to stress levels, 10% difference in stress levels may result in an order of magnitude difference in fatigue life. For this reason, fatigue lives of laminae show much larger variance. However, in all of the optimum designs, the first ply and last ply failure loads are the same. This justifies the use of a first-ply-failure criterion for smooth plates in the present design optimization methodology. However, in the optimization of notched plates, a progressive failure approach may be a better choice.

There are also counter intuitive results. For the loading case, $N_{xx} = 5 \times 10^5$, $N_{yy} = 2.5 \times 10^5$, $N_{xy} = 5 \times 10^5$ N/m, the optimum configuration is $[38_{14}/-52_2]_s$ and its fatigue life is 576,080 cycles (Table 3). When N_{yy} is increased to 7.5×10^5 N/m, the optimum configuration becomes $[34_8/70_8]$ and the fatigue life of the optimum configuration increases to 1.729×10^6 cycles (Table 2). This means that in some cases, if one component of stress is increased, the algorithm may find a design with higher fatigue strength. The reason for this is that increased N_{yy} results in a higher level of stress along the fiber direction in the new optimum configuration, but a lower stress transverse to it due to Poisson's effect. In other words, when N_{yy} is increased from 2.5×10^5 to 7.5×10^5 N/m, the more damaging stress component decreases in the optimal design, while the stress components against which the laminate is much stronger increase.

If, instead of optimal laminates, quasi-isotropic laminates having the same thickness are used under the same conditions, most of them fail at the first load cycle. The failure indices of the quasi-isotropic laminates are much larger than that of the optimal laminates.

Optimum designs with contiguity and angle difference constraints

Some of the laminate designs given in Tables 1 to 3 can be reconfigured to satisfy the constraints; for example $[34_8/70_8]_s$ can be rearranged as $[34_2/70_2/34_2/70_2/34_2/70_2/34_2/70_2]_s$ so that the laminae do not contain more than two plies and the angles between the neighboring laminae do not exceed 45°. On the other hand, for some loading cases, constraints need to be imposed. Otherwise, growth of transverse cracks and edge delaminations, which are the factors not accounted for by the fatigue assessment model, may significantly reduce the fatigue life of the laminate. Table 4 presents the constrained optimization results. The failure indices of the optimum laminate designs satisfying the constraints are not much lower than that of the designs obtained with unconstraint optimization; some of them have even higher values.

Conclusions

In this study, an optimization procedure, a variant of SA, is proposed to find the globally optimum designs of composite laminates subjected to given in-plane loads for maximum fatigue life. Fiber orientation angle in each layer is taken as design variable. The fatigue-life prediction models proposed in the literature were examined and compared as to their success in correlating with the experimental data in a wide range of loading conditions and laminate designs. The model proposed by Fawaz and Ellyin[21] accounts for the effect of fiber angle, biaxial loading, and R ratio. For this reason, this model is used to estimate the fatigue lives of the configurations generated during optimization.

The search algorithm turned out to be consistent and reliable in finding the optimum designs. The optimization process was repeated at least 10 times starting from random initial configurations for each loading case. In almost all of these runs, the same optimum design was found even with a large number of optimization variables.

Results are also obtained by imposing contiguity constraint, that means no lamina may contain more than two plies, and the constraint that the difference between two consecutive laminae may not exceed 45°. The fatigue model does not account for these factors affecting fatigue life.

The fatigue strength is correlated with the static strength. Laminates that are stronger against static loading are also stronger against fatigue loading. However, for a number of loading cases, some laminate designs were found to have longer fatigue lives in comparison to some others; but they also have lower safety factors against static failure according to the chosen failure criteria.

Fatigue life was found to be very sensitive to stress level. Increasing the stress level dramatically shortens the fatigue life. Therefore, one should be more cautious in designing structures against fatigue failure.

The first-ply and last-ply failure loads are found to be the same for all of the optimum designs. This justifies the use of the first-ply failure approach in the present study.

In some loading cases, the optimal designs can be counter intuitive. Sometimes, when one component of loading is increased, it is possible to find a laminate design with a longer fatigue life. Therefore, a design process for composite materials should not be based on intuition or experience.

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Conflict of interest

None declared.

References

1. Hahn HT and Kim RY. Fatigue behavior of composite laminate. *J Compos Mater* 1976; 10: 156–180.
2. Adali S. Optimization of fibre reinforced composite laminates subject to fatigue loading. In: Marshall IH (ed.) *Composite structures*. London, UK: Elsevier Applied Science, 1986, pp.43–55.
3. Walker M. A method for optimally designing laminated plates subject to fatigue loads for minimum weight using a cumulative damage constraint. *Compos Struct* 2000; 48: 213–218.
4. Harris B. *Fatigue in composites: science and technology of the fatigue response of fibre-reinforced plastics*, 2nd ed. Cambridge, UK: Wood-head Publishing Ltd, 2008.
5. Sihn S and Park JW. MAE: an integrated design tool for failure and life prediction of composites. *J Compos Mater* 2008; 42: 1967–1988.
6. Reifsnider KL. *Fatigue of composite materials*. London: Elsevier Science Ltd, 1991.

7. Quaresimin M, Susmel L and Talreja R. Fatigue behaviour and life assessment of composite laminates under multiaxial loadings. *Int J Fatigue* 2010; 32: 2–16.
8. Caprino G. Predicting fatigue life of composite laminates subjected to tension-tension fatigue. *J Compos Mater* 2000; 34: 1334–1355.
9. Rotem A and Nelson HG. Failure of a laminated composite under tension-compression fatigue loading. *Compos Sci Technol* 1989; 36: 45–62.
10. Wu WF, Lee LJ and Choi ST. A study of fatigue damage and fatigue life of composite laminates. *J Compos Mater* 1996; 30: 123–137.
11. Smith EW. *Cyclic biaxial deformation and failure of a glass fibre-reinforced composites*. PhD Thesis, Cambridge University, England, 1976.
12. Sims DF and Brogdon VH. Fatigue behavior of composites under different loading modes. In: Reifsnider KL and Lauraitis KN (eds) *Fatigue of filamentary composite materials, ASTM Special Technical Publication No: 636*. Philadelphia: American Society for Testing and Materials, 1977, pp.185–205.
13. Rotem A. Fatigue failure of multidirectional laminate. *AIAA J* 1979; 17: 271–277.
14. Rotem A and Hashin Z. Fatigue failure of angle ply laminates. *AIAA J* 1976; 14: 868–872.
15. Shokrieh MM and Lessard LB. Multiaxial fatigue behaviour of unidirectional plies based on uniaxial fatigue experiments-I. modeling. *Int J Fatigue* 1997; 19: 201–207.
16. Bathias C. An engineering point of view about fatigue of polymer matrix composite materials. *Int J Fatigue* 2006; 28: 1094–1099.
17. Hashin Z. Failure criteria for unidirectional fiber composites. *J Appl Mech* 1980; 47: 329–335.
18. Kere P, Lyly M and Koski J. Using multicriterion optimization for strength design of composite laminates. *Compos Struct* 2003; 62: 329–333.
19. Harik VM and Bogetti TA. Low cycle fatigue of composite laminates: a damage-mode-sensitive model. *J Compos Mater* 2003; 37: 597–610.
20. Fawaz Z and Ellyin F. A new methodology for the prediction of fatigue failure in multidirectional fiber-reinforced laminates. *Compos Sci Technol* 1995; 53: 47–55.
21. Fawaz Z and Ellyin F. Fatigue failure model for fiber-reinforced materials under general loading conditions. *J Compos Mater* 1994; 28: 1432–1451.
22. Nijssen RPL. *Fatigue life prediction and strength degradation of wind turbine rotor blade composites*. Delft, Netherlands: Knowledge centre wind turbine materials and constructions, 2006.
23. Noda J, Nakada M and Miyano Y. Fatigue life prediction under variable cyclic loading based on statistical linear cumulative damage rule for CFRP laminates. *J Reinf Plast Compos* 2007; 26: 665–680.
24. Shokrieh MM and Lessard LB. Progressive fatigue damage modeling of composite materials, part I: modeling. *J Compos Mater* 2000; 34: 1056–1080.
25. Shokrieh MM and Lessard LB. Progressive fatigue damage modeling of composite materials, part II: material characterization and model verification. *J Compos Mater* 2000; 34: 1081–1115.
26. Liu Y and Mahadevan S. Probabilistic fatigue life prediction of multidirectional composite laminates. *Compos Struct* 2008; 69: 11–19.
27. Wu FQ and Yao WX. A model of the fatigue life distribution of composite laminates based on their static strength distribution. *Chin J Aeronaut* 2008; 21: 241–246.
28. Kassapoglou C. Fatigue life prediction of composite structures under constant amplitude loading. *J Compos Mater* 2007; 41: 2737–2754.
29. Ellyin F and El-Kadi H. A fatigue failure criterion for fiber reinforced composite laminae. *Compos Struct* 1990; 15: 61–74.
30. Epaarachchi JA and Clausen PD. An empirical model for fatigue behavior prediction of glass fibre-reinforced plastic composites for various stress ratios and test frequencies. *Compos A Appl Sci Manuf* 2003; 34: 313–326.
31. Hashin Z and Rotem A. A fatigue failure criterion for fiber reinforced materials. *J Compos Mater* 1973; 7: 448–464.
32. Mouritz AP. A simple fatigue life model for three-dimensional fiber-polymer composites. *J Compos Mater* 2006; 40: 455–469.
33. Mao H and Mahadevan S. Fatigue damage modeling of composite materials. *Compos Struct* 2002; 58: 405–410.
34. Nyman T. Composite fatigue design methodology: a simplified approach. *Compos Struct* 1996; 35: 183–194.
35. Hwang W and Han KS. Fatigue of composites-fatigue modulus concept and life prediction. *J Compos Mater* 1986; 20: 154–165.
36. Vassilopoulos AP and Bedi R. Adaptive neuro-fuzzy inference system in modelling fatigue life of multidirectional composite laminates. *Comput Mater Sci* 2008; 43: 1086–1093.
37. Al-Assadi M, El Kadi H and Deiab IB. Predicting the fatigue life of different composite materials using artificial neural networks. *Appl Compos Mater* 2010; 17: 1–14.
38. Huang ZM. Micromechanical life prediction for composite laminates. *Mech Mater* 2001; 33: 185–199.
39. Rotem A and Nelson HG. Residual strength of composite laminates subjected to tensile-compressive fatigue loading. *J Compos Technol Res* 1990; 12: 76–84.
40. Irisarri FX, Bassir DH, Carrere N, et al. Multiobjective stacking sequence optimization for laminated composite structures. *Compos Sci Technol* 2009; 69: 983–990.
41. Xu L-Y. Influence of stacking sequence on the transverse matrix cracking in continuous fiber crossply laminates. *J Compos Mater* 1995; 29: 1337–1358.
42. Akbulut M and Sonmez FO. Design optimization of laminated composites using a new variant of simulated annealing. *Comput Struct* 2011; 89: 1712–1724.
43. Ali MM, Torn A and Viitanen S. A direct search variant of the simulated annealing algorithm for optimization involving continuous variables. *Comput Oper Res* 2002; 29: 87–102.